

Notation for Structural Dynamics:

ω = undamped natural frequency (circular) = $2\pi f$
 ω_d = damped natural frequency
 ω_o = frequency of forcing function (steady-state)

	<u>Displacement</u>	<u>Velocity</u>	<u>Acceleration</u>		
Absolute	x	\dot{x}	\ddot{x}		
Relative	x^*	\dot{x}^*	\ddot{x}^*		
Ground	x_g	\dot{x}_g	\ddot{x}_g		
				<u>Absolute</u>	<u>Relative</u>
Displacement Response Spectrum				S_D	S_D^*
Velocity Response Spectrum				S_V	S_V^*
Acceleration Response Spectrum				S_A	S_A^*
Pseudo Displacement Response Spectrum				S_{PD}	S_{PD}^*
Pseudo Velocity Response Spectrum				S_{PV}	S_{PV}^*
Pseudo Acceleration Response Spectrum				S_{PA}	S_{PA}^*

Piecewise Constant Recurrence Formulas - Ground Acceleration:

$$x_i^* = e^{-\xi\omega\Delta t_i} \left[x_{i-1}^* \cos\omega_d\Delta t_i + \frac{\dot{x}_{i-1}^* + \xi\omega x_{i-1}^*}{\omega_d} \sin\omega_d\Delta t_i \right] + \frac{\ddot{x}_{gi}}{\omega^2} \left[1 - e^{-\xi\omega\Delta t_i} \left(\cos\omega_d\Delta t_i + \frac{\xi\omega}{\omega_d} \sin\omega_d\Delta t_i \right) \right]$$

$$\frac{\dot{x}_i^*}{\omega_d} = e^{-\xi\omega\Delta t_i} \left[-x_{i-1}^* \sin\omega_d\Delta t_i + \frac{\dot{x}_{i-1}^* + \xi\omega x_{i-1}^*}{\omega_d} \cos\omega_d\Delta t_i - \frac{\xi\omega}{\omega_d} \left(x_{i-1}^* \cos\omega_d\Delta t_i + \frac{\dot{x}_{i-1}^* + \xi\omega x_{i-1}^*}{\omega_d} \sin\omega_d\Delta t_i \right) \right] + \frac{\ddot{x}_{gi}}{\omega^2} e^{-\xi\omega\Delta t_i} \left[1 + \left(\frac{\xi\omega}{\omega_d} \right)^2 \sin\omega_d\Delta t_i \right]$$

$$\ddot{x}_i = -2\xi\omega\dot{x}_i^* - \omega^2 x_i^*$$